NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH, APPLIED SCIENCES AND NATURAL RESOURCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION: Bachelor of Science in Applied Mathematics and Statistics	
QUALIFICATION CODE: 07BSAM	**LEVEL:** 7
COURSE CODE: NUM702S	**COURSE NAME:** NUMERICAL METHODS 2
SESSION: JANUARY 2023	**PAPER:** THEORY
DURATION: 3 HOURS	**MARKS:** 93

SECOND OPPORTUNITY/SUPPLEMENTARY – QUESTION PAPER	
EXAMINER	Dr S.N. NEOSSI NGUETCHUE
MODERATOR:	Prof S.S. MOTSA

INSTRUCTIONS

1. Answer ALL the questions in the booklet provided.
2. Show clearly all the steps used in the calculations. All numerical results must be given using 5 decimals where necessary unless mentioned otherwise.
3. All written work must be done in blue or black ink and sketches must be done in pencil.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 3 PAGES (Including this front page)

Attachments

None

Problem 1 [32 Marks]

1-1. Find the best function in the least-squares sense that fits the following data points and is of the form $f(x) = a\sin(\pi x) + b\cos(\pi x)$: [5]

x	-1	-1/2	0	1/2	1
y	-1	0	1	2	1

1-2. Find the Padé approximation $R_{2,2}(x)$ for $f(x) = \tan(\sqrt{x})/\sqrt{x}$ starting with the MacLaurin expansion

$$f(x) = 1+\frac{x}{3}+\frac{2x^2}{15}+\frac{17x^3}{315}+\frac{62x^4}{2835}+\cdots.$$ [12]

1-3. Use the result in **1-2.** to establish $\tan(x) \approx R_{5,4} = \dfrac{945x - 105x^3 + x^5}{945x - 420x^2 + 15x^4}$. [3]

1-4. Compare the following approximations to $f(x) = \tan(x)$ [12]

$$\text{Taylor: } T_9(x) = 1+\frac{x}{3}+\frac{2x^2}{15}+\frac{17x^3}{315}+\frac{62x^4}{2835}$$
$$\text{Padé: } R_{5,4}(x) \quad \text{(given in 1-3.)}$$

on the interval $[0, 1.4]$ using 8 equally spaced points x_k with $h = 0.2$. Your results should be correct to 7 significant digits.

Problem 2 [25 Marks]

For any non negative interger n the Chebyshev polynomial of the first kind of degree n is defined as

$$T_n(x) = \cos\left[n\cos^{-1}(x)\right], \quad \text{for } x \in [-1, 1].$$

2-1. Use the identity/formula: $\displaystyle\sum_{k=0}^{N} \cos(\varphi + k\alpha) = \frac{\sin\frac{(N+1)\alpha}{2}\cos(\varphi + \frac{N}{2}\alpha)}{\sin\frac{\alpha}{2}}$

to show that: [12]

$$\sum_{k=0}^{N} T_m(x_k)T_n(x_k) = 0, \quad \text{for } m \neq n,$$

where $x_k = \cos\left[\dfrac{(2k+1)\pi}{2(N+1)}\right]$, $0 \le k \le N$, are the roots of T_{N+1}.

2-2. Compute the expressions of the first five Chebyshev polynomials of the first kind T_2, T_3, T_4, T_5 and T_6. [5]

2-3-1. Find $P_6(x)$ the sixth MacLaurin polynomial for $f(x) = xe^x$. [3]

2-3-2. Use Chebyshev economisation to economise $P_6(x)$ once. [5]

Problem 3 [36 Marks]

3-1. Determine the number n so that the composite Simpson's rule for $2n$ subintervals can be used to compute the following integral with an accuracy of 5×10^{-9}. [10]

$$\int_2^7 dx/x.$$

3-2. State the three-point Gaussian Rule for a continuous function f on the interval $[-1, 1]$. [3]

3-3. Use the Composite Simpson's rule with four equal subintervals to approximate the following integral and compare your result with the one obtained when using the three-point Gaussian Rule [10]

$$I = \int_{-1}^{1} (2x^4 + 5)dx.$$

3-4. Was the comparison in **3-3.** predictable? Justify your answer. [3]

3-5. The matrix A and its inverse are A^{-1} are given below

$$A = \begin{bmatrix} 1/2 & -1 \\ -1 & 1 \end{bmatrix}, \qquad A^{-1} = \begin{bmatrix} -2 & -2 \\ -2 & -1 \end{bmatrix}.$$

- Use the power method to find the eigenvalue of the matrix A with the smallest absolute value. Start with the vector $\mathbf{x}^{(0)} = (1, 0)^T$ and perform three iterations. [10]

God bless you !!!